

On the Choice of Obtaining and Disclosing the Common Value in Auctions

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Abstract—This paper introduces a game-theoretic analysis of auction settings where bidders' private values depend on an uncertain common value, and only the auctioneer has the option to purchase information to remove that uncertainty. Here, the auctioneer's mission is to reason about whether to purchase the information and, after purchasing it, whether to disclose it to the bidders. Unlike prior work, here the model assumes that bidders are aware of the auctioneer's option to purchase the external information but not necessarily aware of her decision. Our analysis of the individual revenue-maximizing strategies results in the characterization of a Bayesian Nash equilibrium profile. Our equilibrium's analysis results in several findings including the following non trivial results: (i) the availability of external information may minimize the auctioneer's expected revenue; (ii) using the pricing scheme for expensive information may benefit the auctioneer; (iii) in contrast to traditional results, increasing the number of bidders does not necessarily increase expected revenue.

Keywords-auction; private value; second price bid; social welfare; equilibrium;

I. INTRODUCTION

Recent advances in information technologies support the emergence of dynamic pricing mechanisms as the successors of fixed pricing in electronic marketplaces. The success of dynamic pricing is based on their premise to improve revenue and resource utilization. One important dynamic pricing mechanism, where price emerges from the buyers' (e.g., bidders') willingness to pay, is auctions. The success of online auctions gives rise to the role of agents as facilitators and mediators in electronic marketplaces. Moreover, the fact that some online auction mechanisms require bidders to reason about various aspects of their strategies, resulting in an extended complexity of how to compute their strategies, which further strengthens the need for the development of automated software agents [1]. One key aspect that affects bidding in an auction is the way the agents value the auctioned item.

In this paper we consider settings where the auctioned item is characterized by an uncertain common value, on which bidders' private values are based [2], [3], [4], [5]. For example, consider an auction for the lease of an advertising space in a shopping mall. Here, the common value associated with the advertising space is the shoppers'

traffic (e.g., number of shoppers that visit the shopping mall in a certain period of time). Given that common value's figures, each bidder evaluates the leasing contract differently, taking into account its own private value per-potential-shopper revenue. Another example is the classic oil drilling case [2]. Here, the amount of oil and the depth of its location under the ground are the uncertain common values. However, each bidder's valuation of that oil depends on the stratum to which she needs to drill, as each bidder can have different equipment and drilling technology. Similar arguments favoring this hybrid-value model can be suggested for other classic auction domains, such as the U.S. Federal Communications Commission (FCC) [6] and landing slots at airports. Even the classic painting example for private value can be considered as an example for the hybrid model due to the resell factor [3].

In this paper we investigate the cases where the auctioneer has an option of purchasing information that fully eliminates the uncertainty associated with the common value of the auctioned item. Unlike the auctioneer, the bidders in our model have no such option of purchasing information to remove that uncertainty, however they are aware that the auctioneer has this option. The realism for such an assumption is that the information provider's services may need direct access to the auctioned item or some private information associated with it which obviously cannot be accessed without the auctioneer's permission. Note, however, that the fact that bidders are aware of the information purchasing ability makes the equilibrium analysis much more complicated compared to the case where they are not aware (as investigated in [7]). In such a scenario, a substantial part of the auctioneer's strategy is deciding whether or not to obtain the external information, and, if so, whether or not to disclose it to the bidders. In this scenario, the only way the bidders may be exposed to the information once it is revealed is in case the auctioneer herself decides to reveal it. The problem can thus be modeled as a Stackelberg game where the auctioneer is the leader and the followers are the bidders. This modeling is complicated by the fact that in cases where the auctioneer decides not to disclose the information, the situation is actually modeled as a simultaneous game.

The contributions of this paper are twofold: First, it presents

a cohesive formal analysis of the individual revenue-maximizing strategies of the auctioneer and the bidders in settings where bidders become aware of the information that the auctioneer has only if she chooses to disclose said information, and otherwise do not know with any certainty that the auctioneer actually obtained the information in the first place. The individual revenue-maximizing strategies are used to identify the profile strategies that are in Bayesian Nash equilibrium. Unlike prior work that considers models combining private and common value aspects [3], [4], bidders in our model are not limited to an additive combination of the two, and the effect of the common value on all bidders' valuation is not necessarily positively correlated. One important implication of this difference is that, in contrast to results obtained in prior work [8], [5], always disclosing the common value is not necessarily the preferred choice for the auctioneer. The paper illustrates that a possible equilibrium result is for the auctioneer to purchase the information and disclose it only if it turns out to be a value from a subset of the values that she has already identified as ones that may increase her expected utility once disclosed. Second, the paper uses the equilibrium analysis to illustrate several interesting, non-intuitive properties of the model. One surprising result is that the ability to purchase external information may minimize the auctioneer's expected revenue, in comparison to the case where the information is not available in the first place. This is despite the fact that the auctioneer gets to decide whether or not to purchase the information and what portions of it to disclose to the bidders. In fact, we demonstrate that the auctioneer's expected revenue may be minimized even if the information is available to her for free. Moreover, when comparing settings where the information is available for purchase, there are cases where it is better (either for the auctioneer, the bidders or both sides) that the information is highly priced. Another non-intuitive result that somehow conflicts classic auction theory is that the auctioneer's expected revenue may decrease as the number of bidders increases. Namely, the auctioneer may benefit from the departure of some bidders. We further analyzed the model from a social-planner point of view and show that both taxation and subsidies applied to the information's acquisition may increase social welfare. The taxation aims to discourage the auctioneer from purchasing the information while subsidies aim to encourage such purchases. We conclude that both tools can be useful for improving social welfare and that the decision of which to use is setting dependent. In cases where the information is provided by a self-interested agent that attempts to maximize its own revenue, we show that the auctioneer may find it useful to pay the information provider to vanish from the market. The paper is organized as follows: In Section 2 we describe the model in detail. In Section 3 we analyze the equilibrium and provide a procedure for identifying the Bayesian Nash equilibrium for all participants. In Section 4 we illustrate

specific properties of the model using synthetic settings. In Section 5 we survey related work. We conclude in Section 6.

II. THE MODEL

The model assumes a setting where a single auctioneer offers a single item for sale in a second-price sealed-bid auction to n heterogeneous bidders that are interested in said item. Both the auctioneer and the bidders are assumed to be risk-neutral and fully rational. As common in auction literature, the auctioned item is assumed to have a characteristic X whose value, x , is a-priori unknown both to the auctioneer and the bidders [3], [4]. The only information publicly available with regard to X is the set of possible values it can obtain, denoted X^* , and the probability associated with each value, $Pr(X = x)$ which can also be noted as $p(x)$ ($\sum_{x \in X^*} p(x) = 1$). Each bidder is assumed to be characterized by a private type, T . Bidders' types are assumed to be independent and identically distributed, such that the a-priori probability of any of the bidders being of type $T = t$ is given by $Pr(T = t)$ [3], [4]. A bidder's type defines the way she values the proposed item (i.e., her "private value") for any true value of X . We use the function $V_t(x)$ to define the value for bidders of type $T = t$ in case the characteristic X has a value x . The value of X can thus be seen as a common value in this context, and the function $V_t(x)$ defines the way each bidder of type t 's valuation is affected by this common value, x . Unlike prior work that commonly assume correlation between the ways different bidders' valuations are set given the common value (e.g., linear or symmetric dependency on the common value or a symmetric function of the other bidders' signals [4], [5]), the model in this paper does not imply any restriction on the function $V_t(x)$. Bidders are assumed to know their own types, whereas the auctioneer is assumed to be unfamiliar with each specific bidder's type, however she is acquainted with the discrete probability distribution of bidders' types. The model assumes that the auctioneer can obtain the actual value of X for a payment C (e.g., by purchasing it or by hiring the services of an external information provider), **prior to starting the auction**. If the auctioneer chooses to do so, the value x becomes available only to her, and she can either disclose it to all bidders symmetrically, prior to bidding, or keep it to herself. We assume bidders cannot independently obtain the value of X (not even for a fee), and the only way they can become aware of the true common value is if the auctioneer obtains and discloses it. The realism for such an assumption may be the fact that the information provider's services might need direct access to the auctioned item or some private information associated with it that obviously cannot be

accessed without the auctioneer’s permission.¹ The model assumes that the auctioneer is committed to the value she discloses, therefore bidders are guaranteed that a disclosed value is necessarily the true value of X . If no information regarding the value of X is received from the auctioneer prior to bidding, then the bidders cannot distinguish between not receiving this information due to the auctioneer not purchasing it in the first place, and not receiving it because the auctioneer prefers not to disclose the value she obtained. Both the auctioneer and the bidders are assumed to be self-interested and attempt to maximize their own expected benefit. The auctioneer’s expected benefit is defined as the expected revenue from the auction minus the payment C if choosing to obtain the information. A bidder’s benefit is its valuation of the item minus its payment to the auctioneer (which is the second highest bid) if she wins the auction and zero otherwise. Finally, we assume the existence of a social planner (e.g., a government or a market/platform owner) that is assumed to be aware of the number of bidders in the auction, n , the discrete random variables, X and T , their possible values and their discrete probability distributions. The social planner aims to maximize the “social welfare”, defined as the sum of the expected benefits of all participants. Consequently, she can decide to tax the purchase of information or subsidize the cost of obtaining the information if this will result in an increase in the social welfare. The tax, if used, is assumed to be a positive element in the social welfare and the subsidy is a negative one.

III. ANALYSIS

In this paper we expand our prior work [7] by developing a higher layer of strategy analysis which takes into consideration the critical assumption that bidders are aware of the possibility that the auctioneer can purchase information to eliminate uncertainty. Moreover, in prior work the auctioneer’s task was to identify the strategy that maximizes its own revenue. In contrast, here the introduced assumption of the bidders’ awareness raises the need for equilibrium considerations, as the auctioneer cannot ignore the bidders’ beliefs and strategies any longer. We begin by analyzing the bidders’ and the auctioneer’s individual benefit-maximizing strategies, assuming that the strategies of the other players are fixed. Moreover, this part of the analysis augments the principles outlined in [7], for the case where the bidder’s valuations are defined over a set of discrete values. Based on the benefit-maximizing equations we present an equilibrium analysis. We conclude this section by identifying a Bayesian Nash equilibrium for the participants to use.

¹The possibility that bidders independently obtain the value is beyond the scope of this paper as it involves several aspects of coalition formation and cooperation among self-interested agents that need to be resolved first.

A. Bidders’ Side

Consider a bidder of type t who participates in the auction. We denote by $R^{bidder} = \{r_1, \dots, r_k\} \subset X$ the set of values that the bidders (eventually) believe to be the true values which the auctioneer will be interested in revealing. Namely, in case they believe in $R^{bidder} = \emptyset$, it means that the bidder believes that the auctioneer will not obtain the information in the first place. The complementing set $NR^{bidder} = X^* - R^{bidder}$ represents the values that the bidder believes the auctioneer will not disclose once revealed.

We distinguish between three types of bidders’ responses to different scenarios they face:

- Since we conduct a second-price sealed-bid, the bidder is assumed to bid her expected private value, given her beliefs regarding the true value of X , as in such auctions truth telling is assumed to be the best response strategy. Therefore, if the auctioneer discloses a value x , then the bid of a bidder of type t , denoted $B(t, x)$ is:

$$B(t, x) = V_t(x) \quad (1)$$

It is notable that, in this case, the bidders’ bid is affected only by the value x disclosed by the auctioneer and is not affected by the bidder’s belief, R^{bidder} , whatsoever. This is because even if the auctioneer discloses a value which the bidder was not expecting to be disclosed (i.e., $x \notin R^{bidder}$), this value “overrides” the set R^{bidder} once it has been disclosed (i.e. making R^{bidder} irrelevant). The value x dictates the private value of the bidders, and the problem maps to a standard second-price sealed bid auction.

- If the auctioneer does not disclose any value, then from the bidder’s point of view the common value x is not in R^{bidder} and therefore must belong to the complementary set NR^{bidder} . The bid placed by the bidder in this case, denoted $B(t, \emptyset)$, equals the expected private value, given that $x \in NR^{bidder}$. Formally:

$$B(t, \emptyset) = \sum_{x \in NR^{bidder}} V_t(x) \cdot P^*(x) \quad (2)$$

where $P^*(x)$ is the posterior probability which is updated such that it excludes all possible values that belong to the R^{bidder} . Namely, the probability of having x be the true common value will now be calculated as:

$$P^*(x) = \begin{cases} 0 & \text{if } x \in R^{bidder} \\ \frac{Pr(X=x)}{\sum_{y \in NR^{bidder}} Pr(X=y)} & \text{if } x \in NR^{bidder} \end{cases} \quad (3)$$

One presumable problem with the above analysis is when R^{bidder} includes all possible values of X , i.e.

when the bidder believes that the auctioneer purchases the information and reveals any value obtained. In this case $NR^{bidder} = \emptyset$ and thus (3) do not hold. Later, as part of the equilibrium analysis, we show that this problem is avoided in the equilibrium calculation process.

- The expected benefit of a bidder of type t from participating in the auction, where bidders believe the auctioneer uses R^{bidder} and the auctioneer indeed uses this set, is given by:

$$\begin{aligned}
ER_{bidders} = & \sum_{t \in Type} p(t) \left(\sum_{x \in R^{auc}} p(x) \sum_{B(t',x) < B(t,x)} (n-1)p(t') \right. \\
& p(t'' | s.t. B(t'',x) \leq B(t',x))^{n-2} (B(t,x) - B(t',x)) + \\
& \sum_{x \notin R^{auc}} p(x) \sum_{B(t',\emptyset) < B(t,\emptyset)} (n-1)p(t') \\
& \left. p(t'' | s.t. B(t'',\emptyset) \leq B(t',\emptyset))^{n-2} (B(t,\emptyset) - B(t',\emptyset)) \right) \quad (4)
\end{aligned}$$

Notice that for each type we calculate its revenue by multiplying its revenue according to the second price bid and its probability and eventually sum up the answer to all of the different types.

B. Auctioneer's Side

We now turn to the analysis of the auctioneer's expected benefit, given the strategy she uses and the strategy used by the bidders. The expected benefit of the auctioneer when disclosing the information $X = x$, denoted $ER_{auc}(X = x)$, is equal to the expected second-best bid when the bidders are given x , formally calculated as :

$$\begin{aligned}
ER_{auc}(X = x) = & \sum_{w \in \{B(t,x) | t \in T\}} w \left(n(n-1)(1 - G(w,x)) \right. \\
& \left. (g(w,x))(G(w,x))^{n-2} + \binom{n}{2} (g(w,x))^2 (G(w,x))^{n-2} \right) \quad (5)
\end{aligned}$$

where $g(w,x)$ is the probability that the bid placed by a random bidder equals w , and $G(w,x)$ is the probability that the bid placed by a random bidder equals w or below, if the auctioneer disclosed the value x , respectively. The calculation iterates over all of the possible second-best bid values, assigning for each its probability of being the second-best bid. As we consider discrete distributions, it is possible to have two bidders placing the same bid. For any given bid value, w , we therefore consider the probability of having either: (i) one bidder bidding more than w , $k \in 1 \dots (n-1)$ bidders bidding exactly w and all of the other bidders bidding less than w ; or (ii) $k \in 2 \dots n$ bidders bidding exactly w and all of the others bidding less than w . Notice that (5) also holds for the case where $x = \emptyset$ (in which case bidders

use (2)). The functions $g(w,x)$ and $G(w,x)$ are given by:

$$\begin{aligned}
G(w,x) &= \sum_{B(t,x) \leq w} Pr(T = t) \\
g(w,x) &= \sum_{B(t,x) = w} Pr(T = t) \quad (6)
\end{aligned}$$

Consequently, if the strategy of the auctioneer is R^{auc} (i.e. revealing any value $x \in R^{auc}$ and not revealing any value if $x \notin R^{auc}$), and the bidders' belief is R^{bidder} (used for updating the probability distribution P^*), then the auctioneer's expected benefit, denoted by $ER(R^{auc}, R^{bidder})$, is:

$$\begin{aligned}
ER(R^{auc}, R^{bidder}) = & \sum_{x \in R^{auc}} Pr(X = x) \cdot ER_{Rev}(x) \\
& + \sum_{x \in NR^{auc}} Pr(X = x) \cdot ER_{NR}(R^{bidder}) - C \quad (7)
\end{aligned}$$

with the exception that if the auctioneer uses $R^{auc} = \emptyset$, i.e., decides not to purchase the information, her expected benefit is calculated according to (7) however without subtracting the cost C .

C. Equilibrium Dynamics

The model used in this paper can be considered a variant of a Stackelberg game [9] where the leader is the auctioneer and the followers are the bidders. In such scenarios the leader first commits to a strategy and then the followers selfishly optimize their own best strategy. The analysis is complicated by the fact that when the auctioneer does not disclose the true outcome, the situation is better described as a simultaneous game. Given this hybrid game scenario we hereby provide the best response analysis for identifying a Bayesian Nash Equilibrium.

The equilibrium profile of strategies can be represented by a set R^* where $R^{bidder} = R^{auc} = R^*$ from the auctioneer and the bidders will not have an incentive to deviate. It is notable that since the bidders are using a best-response strategy, they will always update $P(X)$ when receiving information $X = \emptyset$ according to the most recent update regarding the auctioneer's strategy. Since $X = \emptyset$ does not reveal any new information (except for the case where $X = \emptyset$ and the bidders believe $R^{bidder} = X^*$ which is analyzed below), bidders will always use $R^{bidder} = R^*$. The auctioneer, on the other hand, being the leader, may find it beneficial to deviate to a strategy $R^{auc} \neq R^*$, given the bidders' belief R^{bidder} . The auctioneer's deviation from a strategy R^* means purchasing the information and using a set $R^{auc} \neq \emptyset$ if $R^* = \emptyset$. Otherwise, if $R^* \neq \emptyset$, the auctioneer may deviate to not purchasing the information (thus necessarily using $R^{auc} = \emptyset$ or simply using $R^{auc} \neq R^*$ such that $R^{auc} \neq \emptyset$).

To find the equilibrium set R^* one needs to iterate over all possible sets R^* and calculate the auctioneer's expected benefit from deviating to a different set R^{auc} , assuming all bidders are using R^* for their probabilistic update whenever not receiving the true value from the auctioneer. If the auctioneer benefits by deviating from R^* to R^{auc} , then the set R^* is not in equilibrium. The process is captured in the following procedure that verifies that a given solution R^* is in equilibrium:

$\forall R^{auc}$ such that $R^{auc} \neq R^*$ if $\left(ER(R^{auc}, R^*) > ER(R^*, R^*)\right)$

return (R^* is not in equilibrium)

where $ER(R^{auc}, R^*)$ is the expected benefit of the auctioneer from using strategy R^{auc} while the bidders believe the strategy she uses is R^* , calculated according to (7) or its variant that does not involve subtracting C (in case $R^{auc} = \emptyset$). Theoretically, it is possible to have multi-equilibria. The determination of which equilibrium will be used in this case is beyond the scope of this paper, though it is likely that it will be the one associated with the maximum expected benefit for the auctioneer, because the auctioneer is the first to move in this game.

The only exception, as discussed above, is when $X = \emptyset$ and the bidders believe that $R^{bidder} = X^*$. In this case the bidders will recognize the fact that their belief is wrong and will update R^{bidder} accordingly. Therefore, a solution of the type ($R^{bidder} = X^*, R^{auc} \neq X^*$) is inapplicable. Instead, because of the buyer's ability to identify the auctioneer's deviation from $R^{auc} = X^*$, and since both the auctioneer and bidders are fully rational and computationally unbounded, both sides will deviate to a strategy R' that maximizes the auctioneer's expected revenue.

Table I illustrates the general solution space in the form of a bi-dimensional matrix, where the rows are the auctioneer's possible strategies and the columns are the bidders' beliefs. Each cell in the matrix contains the expected revenue of the auctioneer given her strategy (determined by the row) and the bidders' beliefs (determined by the column). The matrix is of size $2^n * 2^n$ since any possible value of X can be either disclosed or revealed. The first row, in which $R^{auc} = \emptyset$, is the only row where the cost C does not need to be subtracted from the auctioneer's expected revenue, because if she choose not to reveal any information, the dominating strategy for the auctioneer is to not purchase the information in the first place (see discussion above). The last row represents the case where the auctioneer always reveals the true outcome of X ($R^{auc} = \{v_1, v_2, \dots, v_n\}$). In this case, the auctioneer's expected revenue is

the same regardless of the bidders' belief (i.e., $ER(\{v_1, v_2, \dots, v_n\}, R_i) = ER(\{v_1, v_2, \dots, v_n\}, R_j) \forall R_i \neq R_j$). This is because in this case the bidders will not be required to perform any probabilistic update and will always bid based on the value revealed, using (1).

A strategy R^* that is used both by the auctioneer and the bidders, that yields the auctioneer an expected revenue greater than or equal to the expected revenue if switching to R^{auc} , is necessarily in equilibrium. The characterization of such a strategy within the context of the matrix is a value on the diagonal that is greater than or equal to any other value in its column. One exception for this is the solution where the auctioneer purchases the information and discloses any value received ($R^{auc} = R^{bidder} = \{v_1, v_2, \dots, v_n\}$). In this case, the use of any other strategy $R^{auc'} \neq \{v_1, v_2, \dots, v_n\}$ results in inconsistency to the bidders, whenever the true outcome is of a value that the auctioneer does not reveal according to $R^{auc'}$. Bidders in this case, as discussed above, will deviate to a strategy R'' such that $R'' = \operatorname{argmax}_R \{ER^{auc'}(R, R)\}$.

Theorem 1: The expected revenue of the auctioneer when deviating from (X^*, X^*) to $R^{auc} \subset X^*$, which maximizes its expected revenue, is equal to her expected revenue when both sides use (R^{auc}, R^{auc}) in the first place.

Proof: For any $x \in R^{auc}$ the expected revenue of the auctioneer is identical in (R^{auc}, R^{auc}) and (X^*, R^{auc}) . For any $x \notin R^{auc}$, bidders will be using R^{auc} as well, as discussed above. Hence, based on (3) and (2), the result will be identical. ■

The immediate implication of Theorem 1 is that if the auctioneer's expected benefit when using (X^*, X^*) is greater than or equal to the expected revenue from using (R^{auc}, R^{auc}) for any $R^{auc} \subset X^*$, then (X^*, X^*) is an equilibrium set of strategies.

The solution space characterization illustrated in table I enables highlighting several properties of the equilibrium in our case. For example, Lemma 2 proposes that if that we found two or more strategies that are in equilibrium and one of those strategies is "revealing all of the information to the bidders", then this strategy expected revenue to the auctioneer is the smallest of all of the equilibrium strategy revenues. This is why this strategy will never be selected by the auctioneer.

Lemma 2: If the set (X^*, X^*) is in equilibrium, and if there are any other sets of strategies in equilibrium, then the expected revenue of the auctioneer in any of the other equilibria is at least the expected revenue when using (X^*, X^*) .

Proof: Assume there is another equilibrium (X^{auc}, X^{auc}) . From the equilibrium condition we

auc strategies/ bidders believes	$R_1^{bidder} = \emptyset$	$R_2^{bidder} = \{v_1\}$...	$R_{2N}^{bidder} = \{v_1, v_2, \dots, v_{n-1}, v_n\}$
$R_1^{auc} = \emptyset$	$ER(R_1^{auc}, R_1^{bidder})$	$ER(R_1^{auc}, R_2^{bidder})$...	NA
$R_2^{auc} = \{v_1\}$	$ER(R_2^{auc}, R_1^{bidder}) - C$	$ER(R_2^{auc}, R_2^{bidder}) - C$...	NA
...	NA
$R_{2N}^{auc} = \{v_1, v_2, \dots, v_{n-1}, v_n\}$	$ER(R_{2N}^{auc}, R_1^{bidder}) - C$	$ER(R_{2N}^{auc}, R_2^{bidder}) - C$...	$ER(R_{2N}^{auc}, R_{2N}^{bidder}) - C$

Table I: The model’s solution space (rows represent the auctioneer’s strategy and columns represent bidder’s belief). The term in each cell is the auctioneer’s expected revenue.

obtain $ER(X^*, X^{auc}) \leq ER(X^{auc}, X^{auc})$. Since the auctioneer’s expected revenue when using a strategy $R^{auc} = X^*$ is the same regardless of the bidders’ belief (last row in Table I), we obtain that $ER(X^*, X^*) = ER(X^*, X^{auc}) \leq ER(X^{auc}, X^{auc})$. ■

D. Equilibrium Structure

Based on the equilibrium analysis that was introduced in the former section, we now analyze the effect of the cost of obtaining the information (C) on the equilibrium structure.

Proposition 1: A strategy equilibrium’s structure can be either:

- (\emptyset, \emptyset) , in which case the auctioneer does not purchase the information regardless of its cost.
- (R, R) whenever $C < C^*$ and (\emptyset, \emptyset) when $C \geq C^*$, in which case there is a threshold cost C^* , below which the auctioneer purchases the information and does not purchase if above C^* .

Proof: The proof is based on the fact that once the information is purchased, the payment paid for obtaining it becomes a sunk cost and does not affect the decision of what information should be disclosed. The auctioneer should therefore separately calculate the value of information and then compare it with the cost of obtaining the information in order to decide whether to purchase it. Therefore, there is at most one transition point in the decision of purchasing the information — for small costs the auctioneer may purchase the information, however as the cost of purchase increases, at some critical cost the auctioneer will switch to a strategy $R^{auc} = \emptyset$. The case where $R^{auc} = \emptyset$ regardless of C is when the auctioneer finds it most beneficial never to disclose the information, i.e., even if the information can be obtained for free. ■

Based on Proposition 1 we can now distinguish between three possible behaviors of the auctioneer’s expected revenue as a function of the cost of purchasing the information (see Figure 1). Sketch (a) in the figure is correlated with the case where the auctioneer does not purchase the information, regardless of its cost. Sketches (b) and (c) relate to the case where the decision is based on the threshold C^* . It is notable that the transition to not purchasing the information is associated with an increase in the auctioneer’s expected revenue however never with a decrease. The increase is explained by settings where the favorable solution (to the

auctioneer) of not purchasing the information becomes stable only when the cost is high enough to preclude a deviation to purchasing the information. A decrease will never occur, as the auctioneer will always have an incentive to keep purchasing the information in such cases.

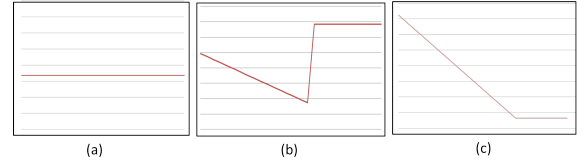


Figure 1: A graphic description of the auctioneer “expected revenue” function.

Finally, Figure 1 facilitates the understanding of information pricing when the information is provided by a self-interested agent. In such a case, the information provider, aiming to maximize its own expected revenue, will choose to set a price tag C^* to the information it provides, as this will guarantee that the auctioneer will buy the information at the maximum possible price. This, of course, applies to cases (b) and (c) in the figure.

IV. NUMERICAL ILLUSTRATIONS AND MARKET DESIGN

Using the equilibrium analysis we described above, in this section we show several interesting results. Among others, we show that the information availability has a great impact on the players’ strategy in equilibrium and inherently affects the outcomes of the players. Moreover, we characterize the scenarios where a social planner may interfere with the game by establishing tax or subsidies as required to lead the market to the best performance.

Since the goal of this section is primarily illustrative, it uses abstract synthetic settings where different bidders’ types are arbitrarily assigned their private value for any common value. In general, all of the examples used are based on three types of bidders, with an equal probability, and three different possible common values which can obtain any value greater than 1 (to which each bidder’s type assigns a different private value). It is notable that the fact that there are three possible bidder types does not constrain the number of bidders available in any way.

We begin by illustrating the effect of some of the model parameters on the different players’ expected revenue. Figure

2 depicts the equilibrium’s expected benefit for the auctioneer, for the bidders and the expected social welfare as a function of the cost of information, given different bidders’ values. The setting used for this figure is captured by Figure 3.

A. Having the option of buying information may diminish the seller’s profit

In figure 2, the auctioneer’s expected benefit first decreases to some cost value in which it exhibits a sharp increase (in a “phase transition”-like pattern) and remains constant afterward as the cost of obtaining the information increases. The transition occurs at a cost $C = 2.75$ (for 7 bidders), where the auctioneer alternates between purchasing the information and not purchasing it. This behavior is correlated with Proposition 1. For any cost lower than 2.75, the equilibrium strategy is to purchase the information and disclose the true outcome if it belongs to the set $\{1, 3\}$ (which does not change, based on Proposition 1). Since the expected revenue from revealing the set $\{1, 3\}$ is fixed, the decrease in the auctioneer’s curve equals the change in C . For any cost greater than $C = 2.75$, the auctioneer avoids purchasing the information, hence her expected benefit is fixed. The information-provider, if self-interested in this case, will determine the cost of information to be the maximum that will result with a purchase by the auctioneer (e.g., the cost set in the case of $N = 7$ is 2.75).

		private value		
		x1	x2	x3
type	p(type)	0.438	0.246	0.316
	p(value)			
1	0.333	62	47	35
2	0.333	83	59	55
3	0.334	92	29	76

Figure 3: Setting of Figure 2

Another interesting, and somehow non-intuitive, observation from Figure 2 is that the expected benefit of the auctioneer given a significantly high cost of purchasing the information is greater than in the case where the information is relatively cheap or even free (e.g., $C = 0$). In such a case, the existence of the option to purchase the information results in a substantial degradation in the auctioneer’s benefit, as she could have benefited far more if such an option did not even exist. The explanation for this result derives from the instability added to the model due to the availability of the information: if the cost of obtaining the information is small, a solution by which the auctioneer does not obtain the information is unstable because of the strong incentive to deviate in order to obtain the information and selectively disclose it. In this kind of setting it could be beneficial for the auctioneer to pay the information provider in order to make her leave the market completely

or, alternatively, to convince her to charge more for the information provided (and publicize the new pricing). For example, in the settings used for Figure 2, there are a number of values for N in which it is beneficial for the auctioneer to pay the information-provider’s profit-maximizing payment just to have her leave the market.

B. Information’s value increases as the number of bidders increases

Another observation based on Figure 2 is that as the number of bidders grows the equilibrium cost threshold by which the auctioneer no longer purchases the information increases. The intuitive explanation for this phenomenon is as follows: the benefit of disclosing the information is in having the bidders bid their true private value. When the number of bidders is relatively small, however, even if a bidder of the type that most values the service bids its true value, based on the common value disclosed, it is possible that the second-best bid will be relatively low and consequently the auctioneer will not profit enough from the information revelation. As the number of agents increases, it is more probable that at least two agents of types that assign a relatively high private value to the disclosed common value are taking part in the auction, and therefore the cost that the auctioneer is willing to pay for the information increases.

C. Having more bidders is not always beneficial

Figure 2 leads to another interesting phenomenon when considering the number of bidders that is benefit-maximizing for the auctioneer as a function of the cost C . In our example (assuming that the auctioneer can pick among the values 3, 5, 7 and 9), the auctioneer will favor having 9 bidders when the cost of obtaining the information is within the range 5.1 – 9.5 and only 7 bidders when within the range 0 – 5.1. This is in contrast to a setting where information cannot be purchased, where the auctioneer will always prefer to have more bidders participate in the auction. This is a surprising result which we did not see in previous literature!

***Are there any references you can bring to “we see in previous literature that more bidders is better” (etti?) not now ***

According to the latest literature, the more bidders there are in an auction, the bigger the expected revenue of the auctioneer. In our model we see the same phenomenon in the cases where the auctioneer decided not to reveal any information to the bidders, but in the cases where the choice was to reveal part of the information to the bidders we are exposed to some surprising results. As we can see from figure 2, when the auctioneer decided not to buy the information and as an outcome didn’t reveal any information to the bidders, her expected revenue when there were 9 bidders was bigger than her expected revenue when there were 7 bidders or less. But when the auctioneer does decide to buy the information and reveal part of it to the bidders

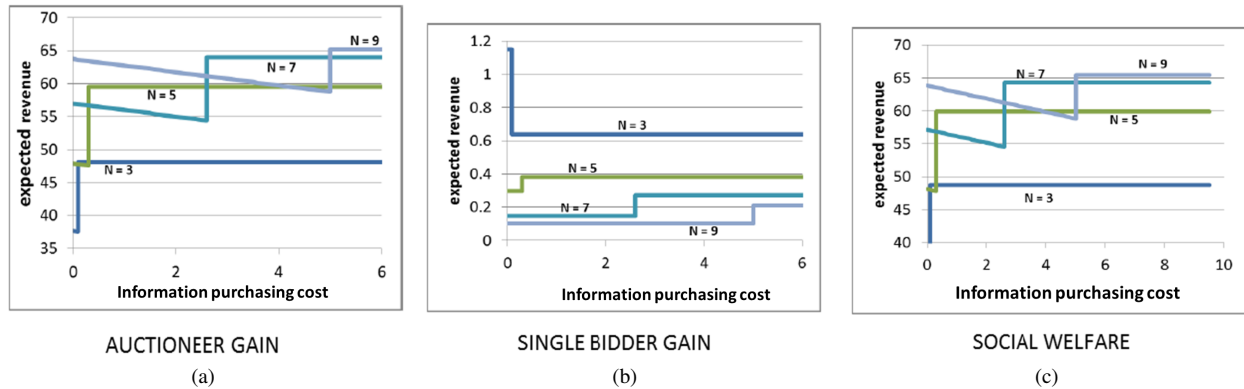


Figure 2: Expected benefit of the auctioneer, the individual buyers’ expected benefit and the social welfare as a function of the information cost and the number of bidders.

(according to the strategy which is in equilibrium), then we can see that in some price ranges it is better for the auctioneer for there to be 7 bidders instead of 9 (or 5 bidders instead of 7). As a consequence, we can conclude that there is not always a preference for a large number of bidders; it all depends on the information which has (or has not) been revealed.

Unlike the auctioneer’s expected benefit, the individual bidder’s expected benefit decreases as the number of bidders increases, as observed from Figure 2. This is explained by the increase in the expected second price bid. Since the buyer’s expected benefit, whenever winning in an auction, is the difference between her private value and the value of the second-best bid, the increase in the latter component results in a decrease of said difference.

V. RELATED WORK

The problem of identifying strategies in order to decide about purchasing information and its strategic disclosure has been studied extensively in various application domains [10], [11]. In particular, one of the most prominent results in auction theory is termed the “Linkage Principle” of Milgrom and Weber [5]. This is when the expected benefit of the auctioneer is enhanced when bidders are provided with more information. This result, however, holds when bidders’ preferences are generally correlated or somehow constrained. For example, Milgrom and Weber [5] consider the private and common values to be independent in their contribution to the overall value for the bidder, and consequently assumes an additive valuation function; Goeree and Offerman [4] assume that all of the bidders’ valuations depend on the common value in the same manner and that each bidder’s valuation is a symmetric function of the other bidders’ signals (***) Etti - is this last sentence true? (***)). In our model, a general valuation function is used, and consequently different results are obtained relating to the usefulness of disclosing information.

Later work observes that information transparency may not be generally optimal [12], [13], [7] *** Dudi: we need a sentence here saying how we’re different from Perry and Krishna. Are they really considering a private-common value model with non-correlated valuations? What is their main result? (Etti)***. Nevertheless, while this prior work does show that a selective information disclosure can be favorable for the auctioneer, the result does not derive from an equilibrium analysis that takes into consideration the strategic behavior of both the bidders and the auctioneer, where bidders may become aware of the auctioneer’s option to purchase the relevant information, as in this paper. In particular, our prior work, which uses a model where the auctioneer has information that it can either disclose or keep secret, assumes asymmetric knowledge regarding the availability of the information, hence the analysis there does not derive from equilibrium considerations. Furthermore, none of the prior works consider market design aspects such as subsidizing or taxing the external information, nor do they consider the information provider as a potentially self-interested agent.

To best of our knowledge, the work most related to ours is the one of Emek et al. [14]. This work investigates the scenario of a publisher who sells ad space to advertisers using a second-price sealed-bid auction in online advertising markets. It is assumed that the auctioneer possesses more accurate information than the advertisers (bidders) and the main research question is which part of the information to disclose to the bidders in order to maximize the publisher’s benefit. Our work is similar to Emek et al. in the sense that both assume: (i) the use of a second-price sealed-bid auction; (ii) the bidders are provided with some a-priori probability regarding the value of the auctioned item; and (iii) a correlated valuation of the auctioned item *** Etti, I thought that our work does not assume correlation in the valuation of bidders. Please advise. **** . Our work

differs, however, in three main aspects. First, our work does not assume that the auctioneer initially has the information, but rather that the auctioneer gets to decide whether or not to purchase that information (which value is a-priori unknown). Consequently, the main questions that our paper deals with are under what conditions to purchase the information and which of the values to disclose. Second, Emek et. al consider an optimization problem (choosing the optimal signaling scheme), while we tackle the problem using Game theory concepts. Namely we show how to find a Bayesian Nash equilibrium. Third, while their work considers the tradeoff between benefit maximization and social welfare, it does not consider external interventions in the form of subsidy and taxation.

Finally, we note that the model where the item's value is a combination of private and common values is sometimes referred to as a *correlated value model* [1]. However, this term is somehow ambiguous and often refers to different model settings as we hereby illustrate. For example, Eso, 2005 [15] studies an auction model with risk-averse bidders where the correlated value stems from the correlation coefficient among the bidders' valuations. A similar model was considered by Wang [16], aiming to determine the preferred selling mechanism (fixed price or the auction) based on the distribution of the potential buyers' valuations. The main finding of that work is that in the case where the buyers' valuations' distribution is sufficiently dispersed or when the object's value is sufficiently high, the auction mechanism is preferred. Many researchers deal with the problem of *uncertainty* in auctions. Most works commonly refer to the uncertainty aspects associated with the bidders (Dyer et al.) [17] consider the case where the bidders are uncertain about the number of bidders participating in the auction, which is often the case in online auctions that apply British type protocols. Parkes [18] and Larson & Sandholm [19] consider the problem where bidders do not know their own private value and need to expend some computational efforts in order to reveal it. They show that there is in fact no correspondence between the classical rational analysis equilibrium and their case where rational bounded agents are considered. Hosam and Khaldoun [20] consider situations where agents are uncertain regarding their task execution, where agents are assumed to have partial control over their resources. None of these works, however, deal with a setting similar to ours, and in particular the question of information acquisition and disclosure is not addressed.

VI. CONCLUSIONS AND FUTURE RESEARCH

The benefits of selective disclosure of information in mixed auction settings have been well established in prior works [7], [14], [12], [13] *** Dudi: I'm not sure about the last two references (Etti)***. The current paper considers the problem in a richer setting, where the availability

of the information to the auctioneer is not trivial, but rather the auctioneer needs to decide whether she is interested in purchasing it. The bidders are aware of this and their strategic behavior is affected by this dilemma. The uniqueness of the analysis derives from the fact that, depending on the auctioneer's choice of revealing the information, the game can be either a Stackelberg game or a simultaneous game. The analysis and the illustrative section that follows it reveal that selective disclosure of information can be a part of the equilibrium solution. Consequently, several market-design-based tools such as subsidy, taxation and payments for individuals in order to leave the market or change their posted prices are considered and demonstrated to be effective.

The equilibrium-based analysis also facilitates the illustration of several interesting and often non-intuitive properties of the model, related to the effect of the different model parameters over the auctioneer's expected-revenue in equilibrium. Some of them are in contrast to those characterizing traditional auction models (e.g., having more bidders participate in the auction is not necessarily the auctioneer's best interest, having the option to purchase the information (at all or at a reduced price) can actually result in a degradation in the auctioneer's expected revenue). These results are attributed to the stability requirement — despite the superiority of the situation for the auctioneer (e.g., with the reduced cost of information, the greater number of bidders), the auctioneer's preferable solution cannot hold since the bidders know that if they act according to it, there is an incentive for the auctioneer to deviate. The case where the information provider is modeled as a self-interested agent is of special interest, as it further enhances strategic dynamics.

Natural extensions of this work are the analysis of settings where only some of the bidders are aware of the fact that the auctioneer has the accurate common value information and the analysis of settings where the auctioneer herself can only obtain a noisy signal for the common value to begin with (e.g., only some values can be eliminated and some uncertainty remains).

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